GEOPHYSICS, VOL. 64, NO. 6 (NOVEMBER-DECEMBER 1999); P. 1806–1815, 14 FIGS.

Multiple attenuation in the parabolic τ-p domain using wavefront characteristics of multiple generating primaries

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ABSTRACT

The problem of multiple attenuation has been solved only partially. One of the most common methods of attenuating multiples is an approach based on the Radon transform. It is commonly accepted that the parabolic Radon transform method is only able to attenuate multiples with significant moveouts. We propose a new 2-D method for attenuation of both surface-related and interbed multiples in the parabolic τ-p domain. The method is based on the prediction of a multiple model from the wavefront characteristics of the primary events. Multiple prediction comprises the following steps:

1) For a given multiple code, the angles of emergence and the radii of wavefront curvatures are estimated for primary reflections for each receiver in the common-shotpoint gather.
2) The intermediate points which compose a specified multiple event are determined for each shot-receiver pair.
3) Traveltimes of the multiples are calculated.

Wavefields within time windows around the predicted traveltime curves may be considered as multiple model traces which we use for multiple attenuation process. Using the predicted multiple traveltimes, we can define the area in the τ-p domain which contains the main energy of the multiple event. Resolution improvement of the parabolic Radon operator can be achieved through a simple multiplication of each sample in the τ-p space by a nonlinear semblance function.

In this work, we follow the idea of defining the multiple reject areas automatically by comparing the energy of the multiple model and the original input data in the τ-p space. We illustrate the usefulness of this algorithm for the attenuation of multiples on both synthetic and real data.

INTRODUCTION

There are two main types of multiples: surface-related multiples, where one or more of the downward reflections occurs at the free surface, and interbed multiples. Multiples are a major form of noise in marine seismic exploration. Until now, the problem of multiple attenuation has been solved only partially. Attenuation methods usually make assumptions about the earth or the character of the source wavelet. These assumptions are often violated, and the efficiency of the attenuation is degraded. Different properties of multiples are used for multiple attenuation: periodicity (Peacock and Treitel, 1969), velocity discrimination (Schneider et al., 1965), coherency (Kneib and Bardan, 1994). The wave-equation-based and inverse-scattering prediction and subtraction methods have attracted much attention in recent years (Wiggins, 1988; Fokkema and Van den Berg, 1990; Verschuur et al., 1992; Weglein et al., 1997; Dragoset and Jericevic, 1998). Noise suppression in the τ-p transform domain was introduced to attenuate multiple energy in data processing. Predictive deconvolution in the linear τ-p domain was investigated by Taner (1980) and Treitel et al. (1982). The parabolic transform was introduced by Hampson (1986) to improve the separation of coherent events in the common-midpoint (CMP) domain. Hampson's transform can be considered as the parabolic τ-p transform. Hampson applied the parabolic transform to NMO-corrected gathers, since the residual moveout of reflections may be approximated by a parabola. A similar transformation was proposed by Foster and Mosher (1992). Yilmaz (1989) introduced a t^2-stretching of the time axis. This transformation maps reflection hyperbolas to parabolas. A high-resolution Radon transform algorithm was introduced by Sacchi and Ulrych (1995).

Manuscript received by the Editor May 12, 1997; revised manuscript received January 28, 1999.
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In the parabolic τ-p domain, the separation of multiples and primaries is better than in t-x, f-k, or linear τ-p space (Thorson and Claerbout, 1985; Hampson, 1986; Yilmaz, 1989, Foster and Mosher, 1992; Zhou and Greenhalgh, 1994). The multiples are attenuated in the parabolic τ-p domain usually by filtering/muting a suitable corridor over the τ-p gathers.

In order to define this corridor, the multiple model needs to be predicted. For surface-related multiples, Zhou and Greenhalgh (1996) computed the multiple model in the parabolic τ-p domain by wave-equation extrapolation. Kelamis et al. (1990) proposed an alternative scheme whereby the Radon domain multiples are inversely mapped and then subtracted from the original records, yielding primary gathers. In the present work, we propose a new method for attenuation of both surface-related and interbed multiples in the parabolic τ-p domain. The method is based on prediction of a multiple model for an arbitrary 2-D subsurface from the wavefront characteristics of the primary events (Keydar et al., 1996). The first part of this paper describes the prediction scheme with the assumptions required for successful multiple attenuation. In the second part of the paper, we propose two schemes for multiple subtraction in a τ-p domain. Numerical examples as well as a real data example are used to illustrate the proposed method.

MULTIPLE PREDICTION

Multiple suppression is based on multiple model traces prediction. Different multiple attenuation technologies based on the inverse scattering theory and wave equation are used mainly to predict multiples in the marine water layer or the weathering layer. They usually require considerable computer time and fail in peg-leg and interbed multiple prediction. In this paper, a method using wavefront characteristics of primary reflections for multiple prediction has been used (Keydar et al., 1998). The algorithm is based on a simple but powerful idea: the timing of any multiple event (surface related or interbed) consists of segments that are primary events. We shall illustrate this method on the interbed multiple example.

First, consider the kinematic properties of multiples. Figure 1 shows a layered model of the subsurface and a multiple event, $A_S C_1 B_1 C_2 A_r$, for the wave emitted from the source located at point A_S, reflected twice from the nth interface (reflections points C_1 and C_2) and once from the $(n-1)$th interface (reflection point B_1) and emerging at receiver A_r. (The overburden model and ray trajectories through the overburden are shown as dashed lines in Figure 1). From the ray path diagram we see that the interbed multiple $A_S C_1 B_1 C_2 A_r$, can be represented as a combination of three primary events reflected from the interfaces $n-1$ and n, namely $A_S C_1 A_m$, $A_n C_2 A_r$, and $A_n B_1 A_m$. The traveltime of this multiple may be described as

$$T_m = T_{p1} + T_{p2} - T_{p3}, \tag{1}$$

where T_m is the traveltime of the multiple $A_S C_1 B_1 C_2 A_r$, T_{p1} is the traveltime of the primary $A_S C_1 A_m$, T_{p2} is the traveltime of the primary $A_n C_2 A_r$, and T_{p3} is the traveltime of the primary $A_n B_1 A_m$. Thus, to predict the multiple for a given source and receiver positions A_S and A_r, we need to find locations of intermediate points A_n and A_m. From expression (1) and Figure 1 it is evident that:

1) The emergence angle β_m is identical for the wave emitted from source A_S, reflected from the interface n at point C_1 and emerging at point A_m, and the wave emitted from source A_n, reflected from interface $n-1$ at point B_1 and recorded at the same point A_m.
2) The emergence angle β_n is identical for the wave emitted from source A_r, reflected from the interface n at point C_2 and emerging at point A_{n1}, and the wave emitted from source A_m, reflected from interface $n-1$ at point B_1 and recorded at the same point A_n.

These conditions ("multiple conditions") are used to determine the segments of primary events generating the multiple. The prediction procedure consists of three steps:

1) The angle of emergence of the primary reflections from multiple generating interfaces is estimated for every trace of each common shot gather.
2) For a given source-receiver location, primary reflections that satisfy the multiple conditions are selected (thereby defining the points A_n and A_m in Figure 1).
3) The arrival times for multiple events are calculated from known primary reflections.

The crucial step in our prediction procedure is the estimation of the angle of emergence of the reflection wavefront. In this case, we are using an algorithm described in detail by Keydar et al. (1998).

Figure 2 shows the estimating scheme: two incident rays are emitted at source point A_0 reflected from an interface S and emerge at points A_k and A_m; β_k is the angle of emergence of the reflected ray at A_k. The reflection wavefront Σ can be approximated in the vicinity of A_k by a fictitious spherical wavefront with radius R_k and the same angle of emergence, β_k. The local moveout correction $\Delta\tau_k$ for the arbitrary kth trace corresponding to a source A_0 and receiver A_k is described as follows:

$$\Delta\tau_k = \frac{\left[\sqrt{\left(R_k^2 + 2R_k \Delta x \sin\beta_k + \Delta x^2\right)} - R_k\right]}{V_0}, \tag{2}$$

where Δ_x is the distance between receivers A_k and A_m, R_k is the radius of the reflected wavefront at point A_k, and V_o is the near-surface velocity which is presumed known.

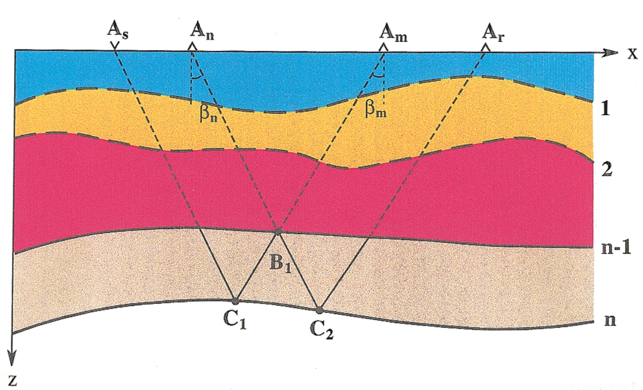

FIG. 1. Kinematic properties of multiples. The interbed multiple $A_S C_1 B_1 C_2 A_r$ can be represented as a combination of three primary events reflected from the interfaces $n-1$ and n, namely $A_S C_1 A_m$, $A_n C_2 A_r$, and $A_n B_1 A_m$.

Unknown parameters β_k and R_k can be estimated using the wave correlation procedure consisting of finding parameters which maximize the semblance correlation measure calculated on the common shot (common receiver) along a time curve defined by equation (2). The two parameter search is performed around the zero-offset time at a specific primary reflection. Knowing β and R for all shot-receiver pairs, we can find the intermediate point (A_n and A_m in Figure 1) using a multiple condition. The multiple traveltime curve can now be calculated from equation (1), assuming that the times for primary reflections are known. Wavefields within time windows around the predicted traveltime curves may be considered as multiple model traces which we use for the multiple attenuation process.

Although 3-D generalization of the proposed multiple prediction scheme is straightforward, two disadvantages should be mentioned here: (1) the need for five parameters (two angles plus three radii of curvature) instead of two parameters as in the 2-D case dramatically increases the cost of the algorithm, and (2) conventional 3-D marine observation systems do not provide sufficient azimuth information and, thus, such an expensive computational scheme as this is ineffective.

MULTIPLE ATTENUATION IN THE PARABOLIC τ-p DOMAIN

Parabolic Radon transformation was introduced by Hampson (1986) to remove long period multiples. The parabolic Radon transform moves parabolic events (after NMO or t^2-stretching) to different areas (points) of the parabolic Radon τ-p space, depending on the events curvature. The parameter p in this paper is equal to the squared slowness of each event. Of particular interest for multiple suppression is the definition of the multiple reject zone. Normally, multiple suppression in the τ-p domain is achieved by zeroing the area where the multiple energy is assumed to be concentrated. It is obvious that the maximum energy of an event in the x-t domain is compressed to the vicinity of a point in the τ-p domain. This point can be defined by parabolic approximation of the multiple time curve predicted by the procedure described above. The energy of a multiple event in the parabolic

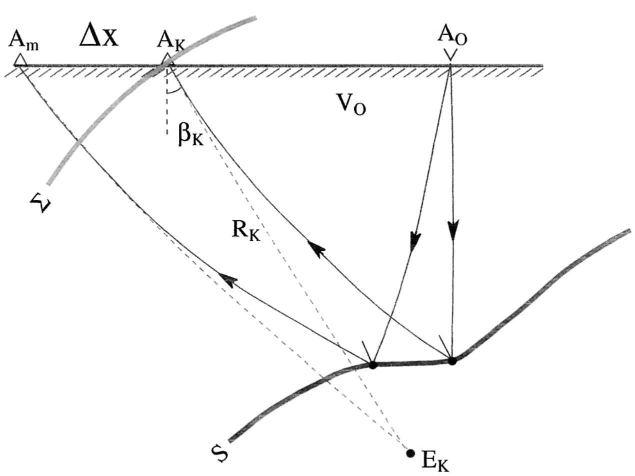

FIG. 2. Ray diagram for estimating the angle of emergence. The wave front Σ emerges at point A_k under the angle β_k. The radius of curvature of this wavefront is R_k. V_o is the near-surface velocity.

τ-p domain is smeared in an area around the central point. This is an area which contains the main energy of the multiple event. However, multiple energy and primaries may not be confined to a limited area. Removing only the area corresponding to the assumed multiple energy may leave strong multiple energy in the filtered section, or the primary energy may be attenuated. Figure 3 illustrates this limitation of the parabolic Radon transform when using its singular value decomposition (SVD) application. The transform of parabolas with 10-ms moveout difference at the far offset in Figure 3a produces essentially no separation of events in the τ-p domain (Figure 3b). In order to enhance the focusing power of the parabolic stack, sparseness criteria may be added to the design of the parabolic-stack operator (Sacchi and Ulrych, 1995). The effect of this nonlinear procedure can be summarized as follows: the nonlinearity produces a model that, while consistent with the data, has minimum structure or maximum sparseness. Resolution improvement of the parabolic Radon operator can be achieved by simple multiplication of each sample in the τ-space by a nonlinear semblance function calculated on the input data along the same parabolic trajectories which we use for obtaining the τ-p transformation. In this case, the semblance function may be considered a weighting function, $w(h)$, in the integral form of velocity stack [Thorson and Claerbout, 1985, Equation their (1)]:

$$u_w(p, \tau) = \int_0^\infty w(h)\, d(h, t)\, dh,$$

where $w(h)$ is a weighting function, $d(h, t)$ denotes the input data, $u_w(p, \tau)$ denotes the output, and h is the offset.

The results of the semblance weighted parabolic stacking (Figure 3c) show improved separation of two events as compared to conventional τ-p transform (Figure 3b). It is worth noting that an exact reconstruction of the (x-t) data in Figure 3a from the τ-p data in Figure 3c is not guaranteed in this case.

To illustrate the way we determine the multiple reject area, let us consider a simple geometrical aspect of the parabolic τ-p transform for a half period of the signal. Denote t^2 as q, and assume that $x_{\min}$ and $x_{\max}$ are the minimum and maximum offsets in the common-shotpoint (CSP)/CMP gather, respectively, and T is the dominant period of the signal in the (x-q) space. Let us also assume that the maximum energy of an event located at zero time q_0 is gathered in the τ-p domain in an area limited in the p direction by $1/v^2_{\min}$ and $1/v^2_{\max}$. It corresponds to the maximum time shift at the largest offset $x_{\max}$, which differs no more than one-quarter period from the optimal time shift as shown in Figure 3. Our aim is to estimate the size and direction of the muting area in the transform space corresponding to the maximum energy of a multiple event.

From Figure 4a we can write $q_{\min} = q_0 + x^2_{\min}/v^2$ and $q_{\max} = q_0 + x^2_{\max}/v^2$. We can also define the limits of the coherent summation as introduced above:

$$\begin{aligned}
q_{\min} - T/4 &= q_{0(\min)} + x^2_{\min}\big/v^2_{\min},\\
q_{\max} + T/4 &= q_{0(\min)} + x^2_{\max}\big/v^2_{\min},\\
q_{\min} + T/4 &= q_{0(\max)} + x^2_{\min}\big/v^2_{\max},\\
q_{\max} - T/4 &= q_{0(\max)} + x^2_{\max}\big/v^2_{\max},
\end{aligned} \qquad (3)$$

where $q_{0(\min)}$ and $q_{0(\max)}$ are the minimum and maximum zero time, respectively, corresponding to the coherent summation.

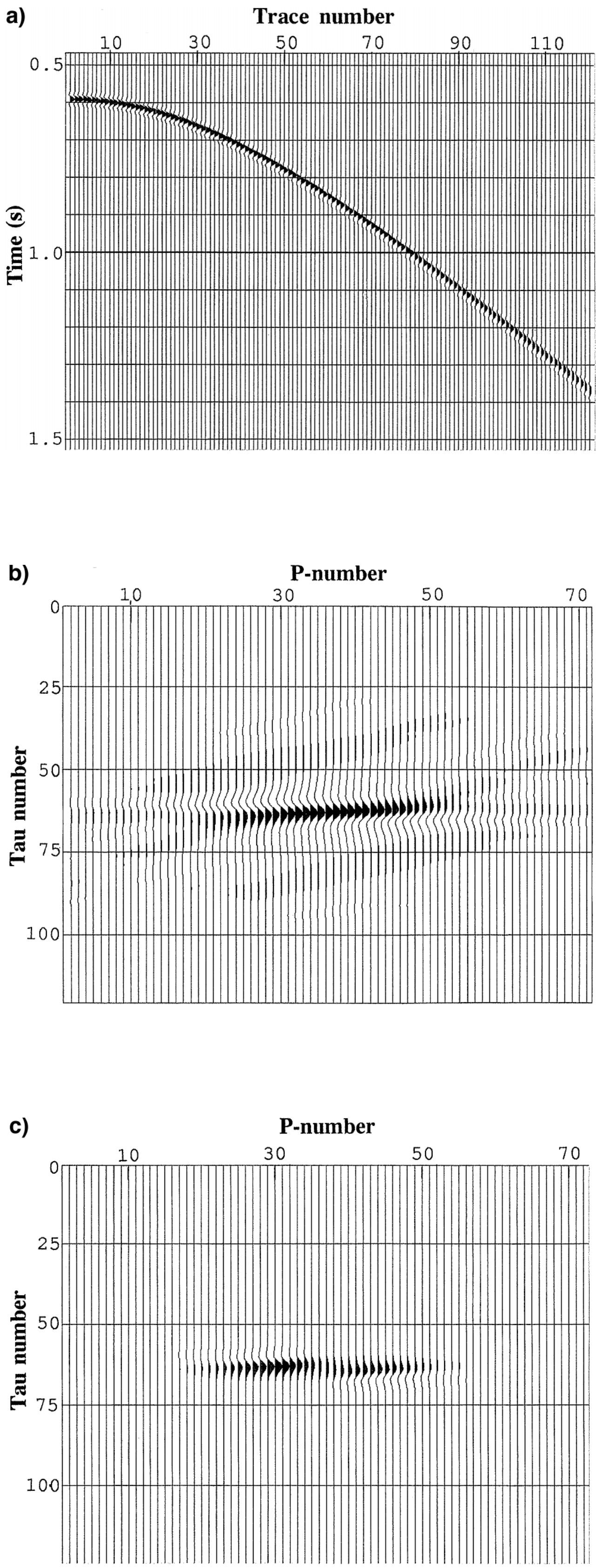

FIG. 3. The parabolic Radon transform: (a) two parabolas with 10 ms moveout different at the far offset; (b) the parabolic τ-p transform when using its SVD application; (c) the semblance weighted parabolic stacking showing improved separation of two events compared to a conventional τ-p transform.

From equations (3), we obtain

$$v_{\min} = \sqrt{\left(x_{\max}^2 - x_{\min}^2\right)/(q_{\max} - q_{\min} + T/2)}$$

and

$$v_{\max} = \sqrt{\left(x_{\max}^2 - x_{\min}^2\right)/(q_{\max} - q_{\min} - T/2)}.$$

Values $\delta p = (1/v_{\min}^2 - 1/v_{\max}^2)$ and $\delta\tau = (q_{0(\max)} - q_{0(\min)}) = x_{\max}^2 \delta p - T/2$ define the length of the multiple reject area in the p and τ directions, respectively. The dip of the area is defined as $\tan(\alpha) = \delta\tau/\delta p$, where $\delta\tau$ and δp are determined from the data as previously mentioned.

Now we can define a muting area for the multiple attenuation filter as an ellipse with long axis a and short axis b (Figure 4b):

$$a = \delta\tau / \sin a,$$
$$b = \sqrt{\frac{T^2/4 - a^2 \sin^2\alpha}{\cos^2\alpha}}. \tag{4}$$

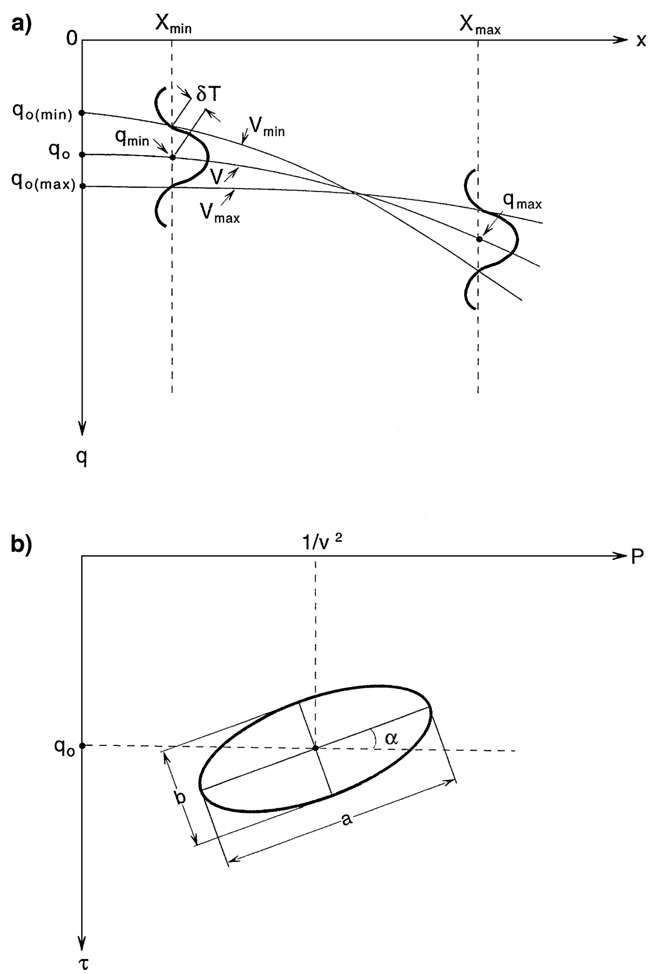

FIG. 4. Determination of the multiple reject area. (a) The maximum energy of event located at zero time q_0 is gathered in the τ-p domain in an area limited in the p-direction by $1/v_{\min}^2$ and $1/v_{\max}^2$. It corresponds to the maximum time shift at the largest offset $x_{\max}$, which differs by no more than 1/4 period from the correct time shift. (b) An ellipse with long axis a, short axes b and dipping angle α serving as the muting area for the multiple attenuation filter.

Bear in mind that the muting ellipse (4) corresponds to a half period of the signal. In practice, we multiply parameter b on a scalar proportional to a wavelet length (in this work we use three half periods).

An elegant alternative design for a multiple suppression filter in the parabolic τ-p domain was proposed by Zhou and Greenhalgh (1996). The multiple reject areas are determined automatically by comparing the energy of the multiple model and the original input data in the τ-p space. In this work, we follow their ideas. The multiple model is constructed by removing the wavefield from the original data in time windows around the traveltimes predicted by the procedure descried above. The multiple model defines where the multiple energy should be attenuated in the original data. By comparing the energy of the multiple model with the input data (multiples + primaries) at each τ-p point, a nonlinear multiple rejection filter is designed. The rejection gain of the filter is sharp but smooth. The gain function of the nonlinear filter in the τ-p domain takes the following form (Zhou and Greenhalgh, 1996):

$$g(\tau, p) = \frac{1}{\sqrt{1 + \left(\dfrac{B(\tau, p)}{\varepsilon 4(\tau, p)}\right)^n}}, \tag{5}$$

where $B(\tau, p)$ is the windowed sum of the absolute amplitude of the pixel centered at (τ, p) on the τ-p version of the predicted multiple model traces, $A(\tau, p)$ is that on the τ-p transformed input data, and n and ε are the multiple rejection parameters. A detailed description of the filter can be found in Zhou and Greenhalgh (1996). Here, we merely note that parameter n is used to control the smoothness of the filter and ε is related to the reflection coefficients of multiple generator interfaces. As explained by Zhou and Greenhalgh (1996), the demultiple filter automatically defines the multiple rejection areas and taper the rejection boundary in the τ-p space. The filter is applied to the original data in the τ-p domain.

SYNTHETIC EXAMPLE

Figure 5 shows a five-layer model used by Zhou and Greenhalgh (1996) for testing their parabolic τ-p filtering technique to suppress multiples. Figure 6a is a CSP gather created for a model calculated using the finite-difference modeling method; a 25-Hz Ricker source wavelet was used in the calculation, and the wave identification is shown in the figure.

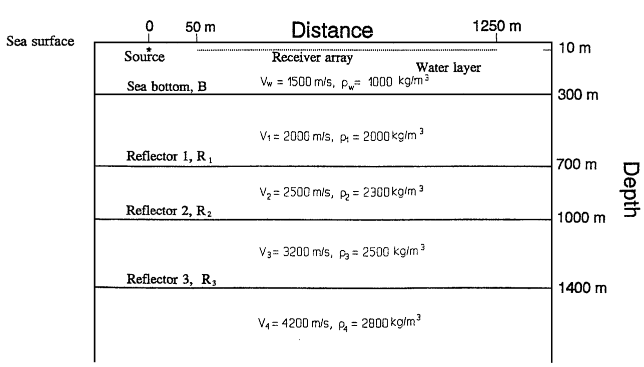

FIG. 5. Five layered model (after Zhou and Greenhalgh, 1996) for testing parabolic τ-p multiple filtering.

The reflection events on the CSP gather are, to a good approximation, hyperbolic. NMO correction or t^2-stretching is necessary to make the hyperbolic events parabolic. We adopted t^2-stretching for this purpose in the synthetic example. Time resampling was used to reduce the aliasing problem near $t = 0$. Figure 6b shows an SVD application of the parabolic τ-p transform (Yilmaz, 1989) after the t^2-stretched seismogram in Figure 6a. It is obvious that the events which interfered with each other in the $(x$-$t)$ domain are well separated in the parabolic $(\tau$-$p)$ space.

The predicted traveltimes for all multiples visible in the input data as obtained by our multiple prediction procedure are shown in Figure 6a. Prediction consists of the following steps.

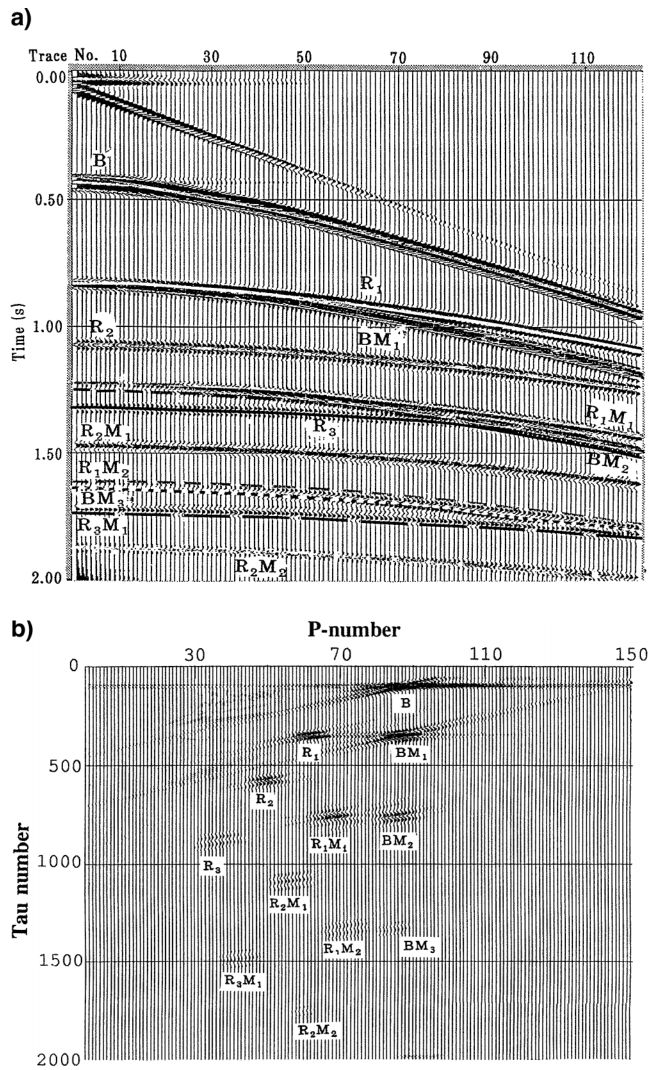

FIG. 6. (a) Synthetic CSP gather calculated for the model in Figure 5 using the FD method. The primary reflectors, B, R_1, R_2, and R_3, are labeled on the gather. The water-bottom multiples of the first, second, third, and fourth orders are labeled BM, BM_2, BM_3, and BM_4, respectively. The symbols R_1M_1, R_1M_2, and R_1M_3 represent the first, second, and third order peg-leg multiples from reflector R_1. The peg-leg multiples from R_2 are R_2M_1 and R_2M_2. Only the first order multiple R_3M_1 from R_3 can be observed. Multiple traveltimes predicted using our prediction procedure are shown. (b) SVD application of the parabolic τ-p transform after the t^2-stretched seismogram in (a).

1) For a given multiple code (in our case water-bottom or surface-related peg-legs), the angles of emergence and the radii of wavefront curvatures were estimated for primary reflections for each receiver in the CSP gather. This was done using the wave correlation procedure and local NMO correction (2).
2) For each shot-receiver pair, the intermediate points participating in the composition of a specified multiple event were determined using the multiple conditions.
3) The traveltimes of the multiples were calculated using expression (1).

Note that higher order multiples can also be predicted using multiple generating primaries. The wavefield removed from the original data in time windows around the predicted traveltimes serve as a multiple model in the suppression procedure (Figure 7a). Figure 7b illustrates the τ-p transform of the multiple model traces. A filter for multiple suppression is devised based on equation (5). The smoothness parameter n of the filter was 8. The multiple rejection parameter ε was 0.3. The results of multiple filtering in the parabolic τ-p domain are displayed in Figure 8a. The inverse parabolic τ-p transform, followed by inverse t^2-stretching, is illustrated in Figure 8b. It is clear that all the multiples are suppressed and the primaries are well preserved. Filtering attenuates multiples equally well for both far and near offset traces.

Figure 9 illustrates an alternative method of multiple attenuation: muting the multiple zone. Figure 9a shows the semblance weighted parabolic τ-p section calculated for the data set shown in Figure 6a. The results show improved separation of the events compared to conventional τ-p transform (Figure 6b). A filter for multiple suppression (muting ellipses) is devised based on the predicted multiple traveltimes and equation (4), and shown in Figure 9a as solid lines. The results of multiple filtering followed by the inverse parabolic τ-p transform and inverse t^2-stretching is illustrated in Figure 9b. [Note again that an exact reconstruction of the (x-t) data from the (τ-p) data in Figure 9b in this case is not guaranteed.]

The basic assumption of both filters for multiple suppression is that the multiples and primaries are separated in the τ-p domain. This requires some differential moveout between the desired primary and undesired multiple. This does not mean,

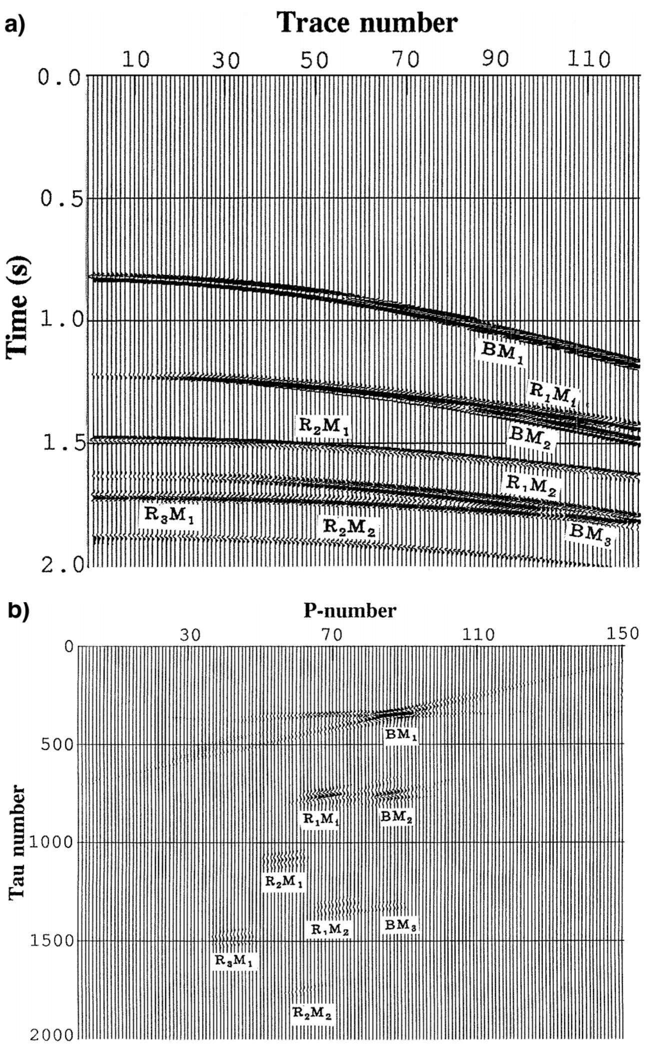

FIG. 7. (a) Multiple model obtained by removing the wavefield from the original data in time windows around predicted traveltimes. (b) The τ-p transform of multiple model traces.

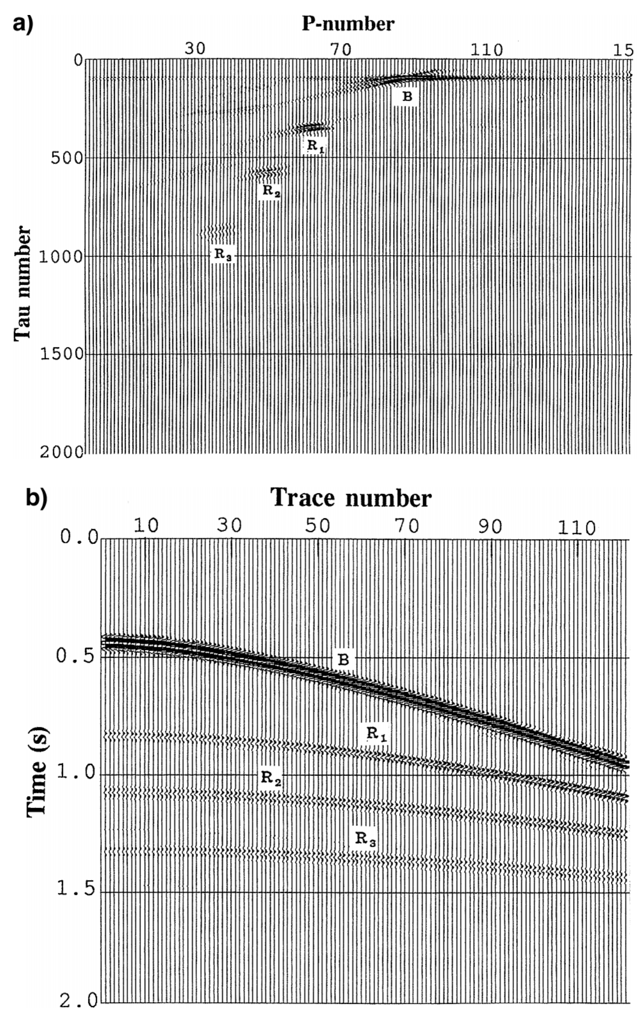

FIG. 8. (a) Results of multiple filtering in the parabolic τ-p domain. (b) Inverse parabolic τ-p transform following by inverse t^2-stretching. All multiples are suppressed and primaries are well preserved.

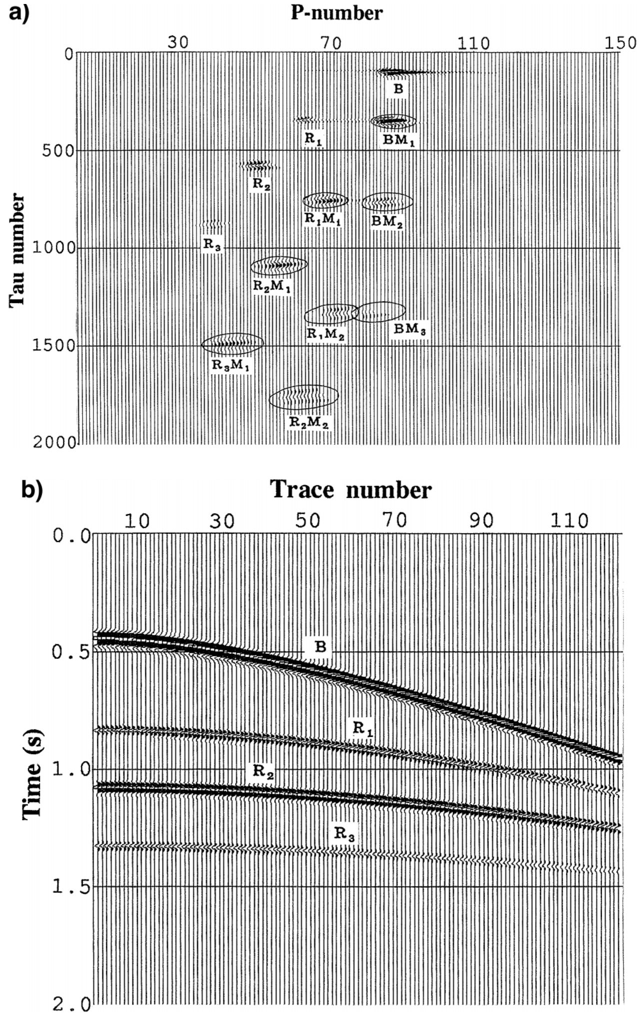

FIG. 9. (a) Semblance weighted parabolic τ-p section calculated for the data set shown in Figure 6a. Results show improved separation of the events compared to conventional τ-p transform (Figure 6b). Filter for multiple suppression (muting ellipses) is shown by solid lines. (b) Results of multiple filtering followed by inverse parabolic τ-p transform and inverse t^2-stretching.

however, that primaries and multiples cannot interfere in the x-t domain; in the semblance weighted parabolic τ-p domain, the separation of multiples and primaries is better than in the t-x or f-k space.

Conventional multiple filtering in the parabolic Radon domain consists of muting the transform space corresponding to the multiples. Selection of the multiples in the Radon domain is usually based on the velocity discrimination between primaries and multiples and is done manually. In practice, the problem is not the velocity discrimination, but rather the definition of the muting area. From this point of view, our method may be considered as a surgical multiple filtering where a major advantage of our approach lies in the fact that the multiple positions in the Radon domain are determined automatically.

REAL DATA EXAMPLE

Following successful experiments with synthetic data, we applied our multiple attenuation scheme to a real data set from a marine seismic field experiment. Figure 10 shows a stacked section and Figure 11 a CMP gather acquired in the Mediterranean Sea off Israel. The water bottom, W, in this area is flat and shallow (about 200 m, and t_0 about 0.4 s); the trace interval in the CMP gather is 25 m and the minimum offset 140 m. The input data contain numerous water reverberations and multiple reflections. We can indicate three water-bottom multiples of several orders (labeled I, II, and III; times about 0.8, 1.2, and 1.6 s) and a strong peg-leg multiple produced by a reflector at about 1.9 s, M, appearing at a time of about 2.3 s (labeled IV). The semblance velocity spectrum of the input CMP gather is shown in Figure 11a. In this case, it is obvious that conventional velocity filtering consisting of muting an area, corresponding to stacking velocities of less than 1500 m, in the parabolic Radon space is inefficient. Multiple locations in the Radon space for the following multiple subtraction/attenuation must be precisely defined. Using information for multiple generating primaries W and M, we predicted the arrival times of three water-bottom and peg-leg multiples for all common shot gathers using our multiple prediction procedure. The wavefield removed from the data in time windows around the predicted traveltimes serves as multiple model traces.

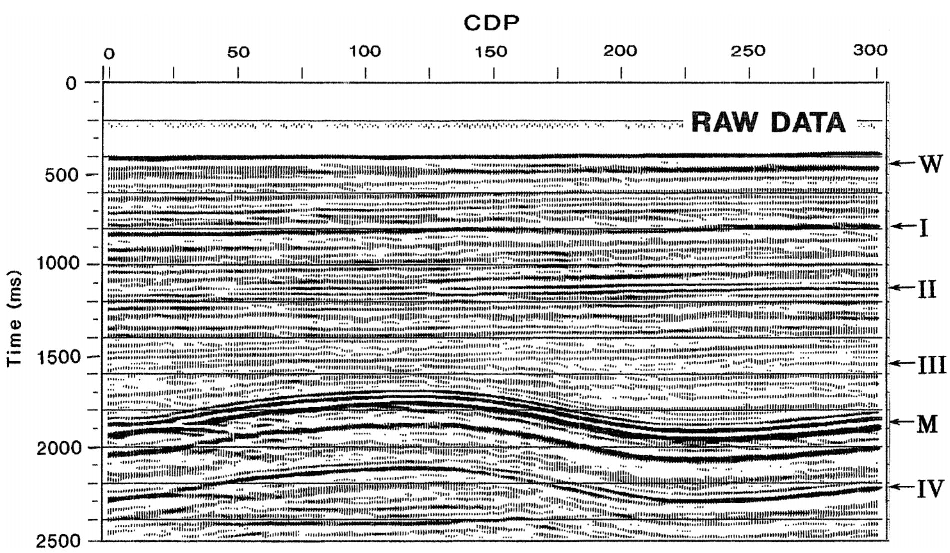

FIG. 10. Stacked section containing numerous water-bottom reverberations and multiple reflections. Three orders of water bottom (labeled I, II, and III), and a strong peg-leg (labeled IV) produced by a reflector M at about 1.9 s are indicated.

After t^2-stretching, the data and the model traces were transformed into the parabolic (τ-p) domain. The total number of p values used in the transform is 150. A 2-D nonlinear filter was designed in accordance with expression (5). The smoothness parameter n of the filter was 8. The multiple rejection parameter ε was 0.3. The resulting stacked section after demultiple is shown in Figure 12. The only processing difference between these sections and those shown in Figure 10 is the application of our multiple attenuation. It is clear that the multiples specified in the filter are, to a great extent, removed from the input data. Some residual multiple energy and reverberations can be observed in the filtered section. In order to illustrate exactly what multiple energy has been removed, we calculated a difference section which is shown in Figure 13. Figure 14 shows a

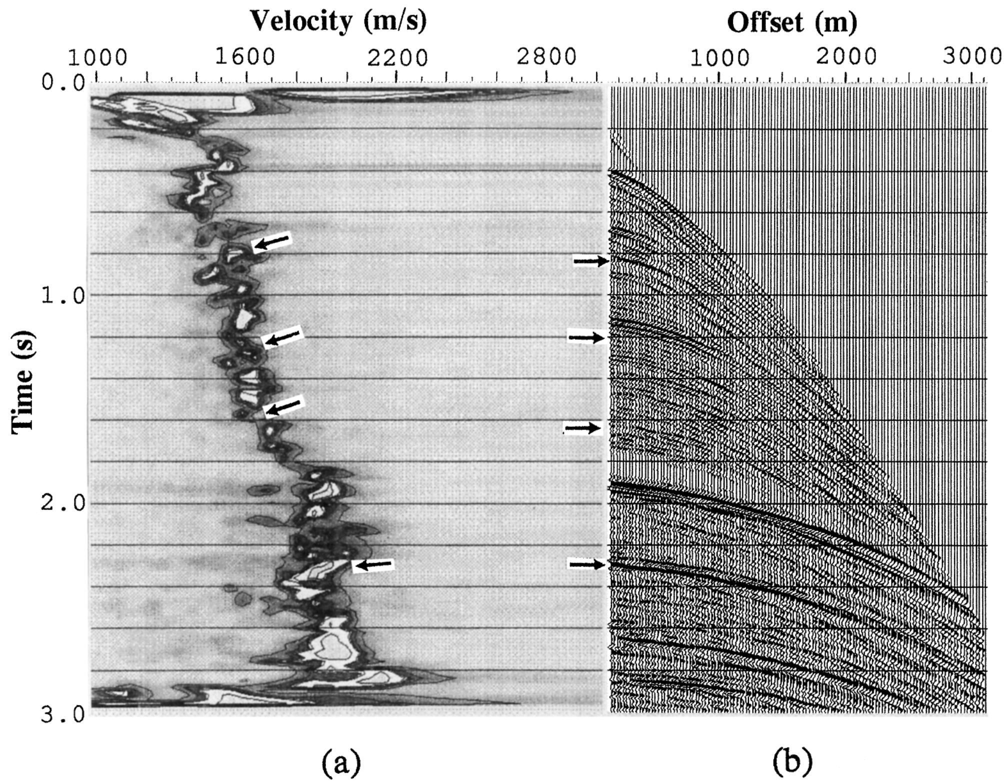

FIG. 11. (a) Semblance velocity spectrum. (b) CMP gather acquired in the Mediterranean Sea off Israel. The trace interval in the CMP gather is 25 m; minimum offset is 140 m. Arrows indicate water-bottom multiples of several orders (times about 0.8, 1.2, and 1.6 s) and a strong peg-leg multiple.

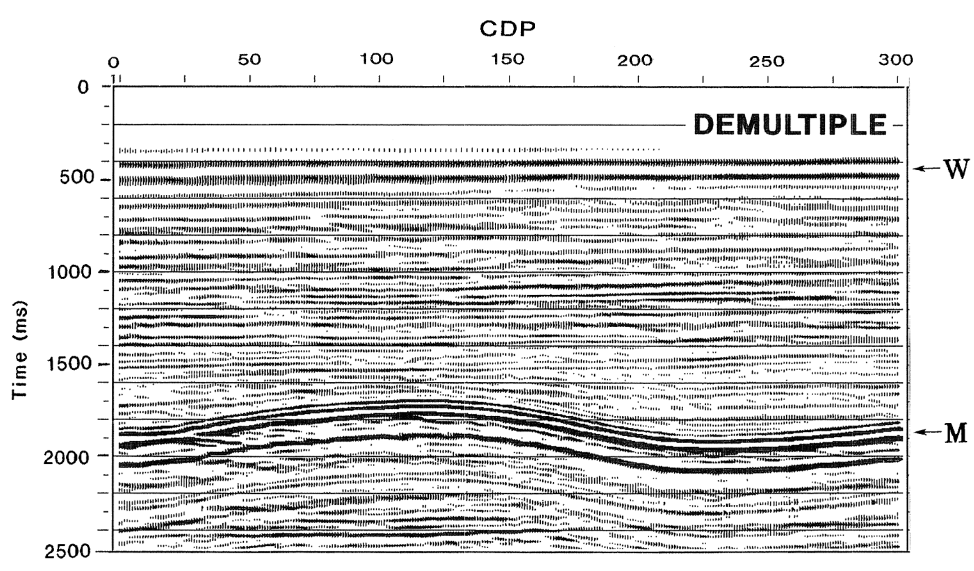

FIG. 12. Stacked section after multiple attenuation. The only processing difference between this and that shown in Figure 10 is the application of our multiple attenuation algorithm.

CMP gather and velocity spectrum after multiple attenuation (compare with Figure 11).

CONCLUSIONS

A method for multiple attenuation in the parabolic τ-p domain is proposed. The demultiple procedure is a combination of the prediction method based on wavefront characteristics of multiple generating primary reflections and the Radon transform–based method for multiple subtraction. The multiple reject areas are determined automatically by zeroing the multiple ellipse in the τ-p domain, or by comparing the energy on the traces of the multiple model and the original input data in the τ-p domain. The advantage of applying the demultiple filter in the parabolic τ-p domain is that the waves are well separated in this domain. Filtering of multiples works well on both near and far offset traces, as long as the hyperbolic approximation of traveltimes holds. The proposed method is adequately

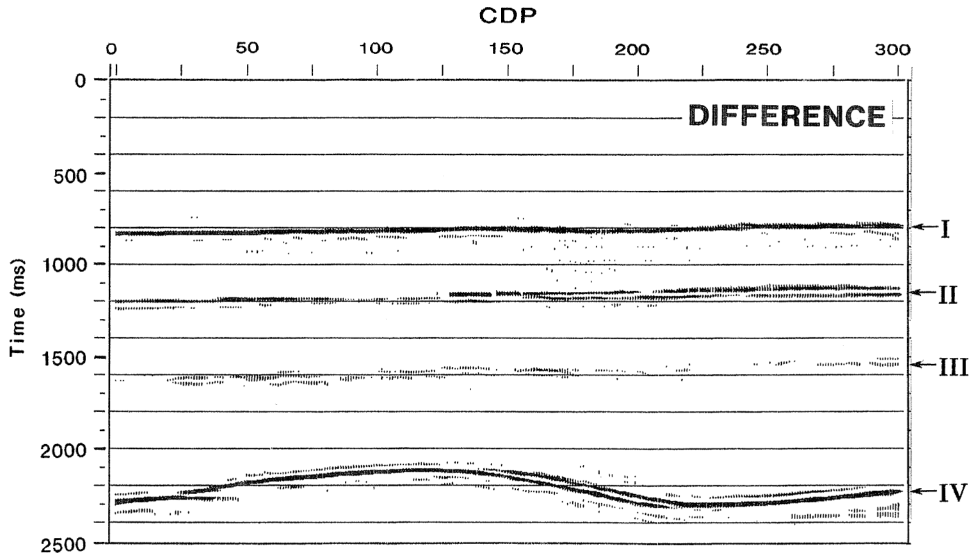

FIG. 13. Differential section illustrating exactly what multiple energy has been removed.

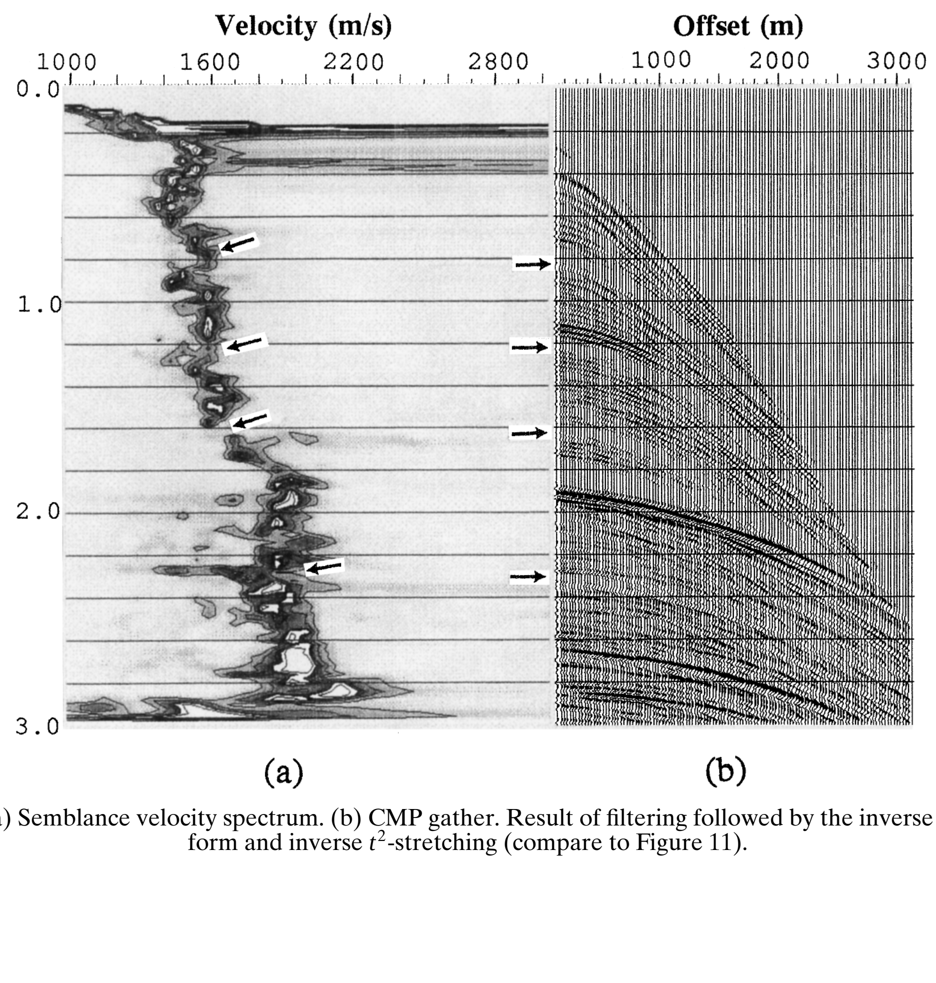

FIG. 14. (a) Semblance velocity spectrum. (b) CMP gather. Result of filtering followed by the inverse τ-p transform and inverse t^2-stretching (compare to Figure 11).

valid for all kinds of multiples: water-bottom, peg-leg, and interbed. The numerical examples demonstrate the effectiveness of the proposed filtering.

ACKNOWLEDGMENT

This work was sponsored by the Earth Science Research Administration of the Israel Ministry of National Infrastructures.

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